Mathematics
Higher Level
Paper 1

Name

Date: ____________________

2 hours

SOLUTION KEY

Instructions to candidates

- Write your name in the box above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all of Section A in the spaces provided.
- Section B: answer all of Section B on the answer sheets provided. Write your name on each answer sheet and attach them to this examination paper.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: analysis and approaches formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[110 marks]**.

worked solutions: 17 pages

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, for example if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A (56 marks)

Answer **all** questions in the boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 5]

Given that $px^3+qx^2-9x+18$ is exactly divisible by $(x+2)(x-3)$, find the value of p and the value of q.

remainder theorem

$x=-2$: $p(-2)^3+q(-2)^2-9(-2)+18=0$

$-8p+4q+36=0 \rightarrow -2p+q=-9$

$x=3$: $p(3)^3+q(3)^2-9(3)+18=0$

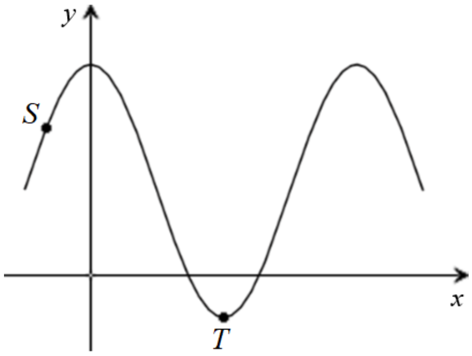
$27p+9q-9=0 \rightarrow 3p+q=1$

$\begin{cases} -2p+q=-9 \\ 3p+q=1 \end{cases}$ subtracting gives $-5p=-10 \rightarrow \underline{\underline{p=2}}$

$3(2)+q=1 \rightarrow \underline{\underline{q=-5}}$

2. [Maximum mark: 6]

The diagram below shows a curve with equation $y = 2 + k\cos x$, defined for $-\frac{\pi}{2} \le x \le \frac{5\pi}{2}$.



The point S lies on the curve and has coordinates $\left(-\frac{\pi}{3}, \frac{7}{2}\right)$. The point T with coordinates (a, b) is the minimum point.

(a) Show that $k = 3$. [2]

(b) Hence, find the value of a and the value of b. [4]

(a) substitute in $\left(-\frac{\pi}{3}, \frac{7}{2}\right)$

$\frac{7}{2} = 2 + k\cos\left(-\frac{\pi}{3}\right)$

$\frac{7}{2} = 2 + k\left(\frac{1}{2}\right)$

$\frac{1}{2}k = \frac{3}{2} \rightarrow k = 3$ Q.E.D.

(b) $y = 2 + 3\cos x \rightarrow \frac{dy}{dx} = -3\sin x = 0$

$\sin x = 0$ find first solution such that $x > 0$

$x = \pi \rightarrow a = \pi$

substituting: $b = 2 + 3\cos(\pi) = 2 + 3(-1) = -1$

thus, $\underline{\underline{a = \pi,\ b = -1}}$

3. [Maximum mark: 6]

A geometric series has a positive common ratio r. The series has a sum to infinity of 9 and the sum of the first two terms is 5. Find the first three terms of the series.

$S_\infty = \frac{u_1}{1-r} = 9 \Rightarrow u_1 = 9 - 9r$

$u_1 + u_1 r = 5$ substituting gives

$9 - 9r + (9 - 9r)r = 5$

$9 - 9r + 9r - 9r^2 = 5$

$9r^2 = 4$

$r^2 = \frac{4}{9}$

$r = \frac{2}{3}$ $(r > 0)$

$u_1 = 9 - 9\left(\frac{2}{3}\right) = 9 - 6 = 3$

$u_2 = 3\left(\frac{2}{3}\right) = 2$

$u_3 = 2\left(\frac{2}{3}\right) = \frac{4}{3}$

first three terms of the series: $3, 2, \frac{4}{3}$

4. [Maximum mark: 5]

The probability density function for a random variable X is given by

$$f(x) = \begin{cases} cxe^{-x}, & 0 \le x \le 1 \\ 0, & \text{otherwise} \end{cases}$$ where c is a real number.

Find the value of c.

$\sum P(X=x) = 1 \rightarrow \int_0^1 f(x)\,dx = 1$

$c\int_0^1 xe^{-x}dx = 1$ find $\int xe^{-x}dx$

integration by parts $u = x \rightarrow du = dx$

$dv = e^{-x}dx \rightarrow v = -e^{-x}$

then $\int xe^{-x}dx = -xe^{-x} + \int e^{-x}dx$

$= -xe^{-x} - e^{-x}$

$c\int_0^1 xe^{-x}dx = c\left[-xe^{-x} - e^{-x}\right]_0^1 = c\left[(-e^{-1} - e^{-1}) - (0-1)\right] = 1$

$c\left(-\frac{2}{e} + 1\right) = 1$

$c\left(\frac{e-2}{e}\right) = 1$

$c = \frac{e}{e-2}$

5. [Maximum mark: 7]

Consider the equation $ax^2+7x-2a=0$ that has two distinct solutions for x.

(a) Given that $x=-3$ is a solution of $ax^2+7x-2a=0$, find the value of a and the other solution for x. [4]

(b) Hence, express $\dfrac{5x-7}{ax^2+7x-2a}$ as the sum of two fractions. [3]

(a) $x=-3$ is a root, therefore $(x-3)$ is a factor of the quadratic.

Hence, $ax^2+7x-2a=(x+3)(\quad ?\quad)=0$. The missing factor must be linear, so

$$ax^2+7x-2a=0=(x+3)(ax+b) \;\Rightarrow\; -2a=3b \;\Rightarrow\; b=-\frac{2}{3}a$$

Then,

$$ax^2+7x-2a=(x+3)\left(ax-\frac{2}{3}a\right) \;\Rightarrow\; ax^2+7x-2a=ax^2+\left(3a-\frac{2}{3}a\right)x-2a$$

Thus,

$$3a-\frac{2}{3}a=7 \;\Rightarrow\; \frac{7}{3}a=7 \;\Rightarrow\; a=3$$

Substituting into the quadratic and factorizing:

$$3x^2+7x-6=(x+3)(3x-2)=0$$

Thus, the other solution to the quadratic equation is $x=\dfrac{2}{3}$

(b) $$\frac{5x-7}{ax^2+7x-2a}=\frac{5x-7}{3x^2+7x-6}=\frac{5x-7}{(x+3)(3x-2)}=\frac{A}{x+3}+\frac{B}{3x-2}$$

Multiply through by $(x+3)(3x-2)$:

$$5x-7=A(3x-2)+B(x+3)$$

Let $x=-3$:

$$5(-3)-7=A(3(-3)-2)+B(-3+3) \;\Rightarrow\; -22=-11A \;\Rightarrow\; A=2$$

Let $x=0$:

$$5(0)-7=2(3(0)-2)+B(0+3) \;\Rightarrow\; -7=-4+3B \;\Rightarrow\; 3B=-3 \;\Rightarrow\; B=-1$$

Hence, $$\frac{5x-7}{ax^2+7x-2a}=\frac{5x-7}{3x^2+7x-6}=\frac{2}{x+3}-\frac{1}{3x-2}$$

6. [Maximum mark: 7]

A curve has equation $4xy - y^2 - x^3 = 0$ for $x > 0, y > 0$. The graph of the curve has a vertical tangent at point R. Find the coordinates of R.

implicit differentiation

$$4y + 4x\frac{dy}{dx} - 2y\frac{dy}{dx} - 3x^2 = 0$$

$$\frac{dy}{dx}(4x - 2y) = 3x^2 - 4y$$

$$\frac{dy}{dx} = \frac{3x^2 - 4y}{4x - 2y}$$

vertical tangent occurs where $\frac{dy}{dx}$ is undefined

denominator of $\frac{dy}{dx} = 0$: $4x - 2y = 0 \Rightarrow y = 2x$

substitute into equation for curve

$$4x(2x) - (2x)^2 - x^3 = 0$$

$$4x^2 - x^3 = 0$$

$x^3 = 4x^2$ since $x > 0$, can divide by x^2

$x = 4$

$y = 2(4) = 8$

coordinates of R: $\underline{\underline{(4, 8)}}$

7. [Maximum mark: 7]

Prove by mathematical induction that $2^n > 2n+1$ for all $n \geq 3, n \in \mathbb{Z}$.

show true for $n=3$

$2^3 > 2(3)+1 \rightarrow 8 > 7$ thus, true for $n=3$

assume statement to be true for a specific value of n, call it k

thus, $2^k > 2k+1$

show that it must follow that statement is true for $n=k+1$

hence, show that $2^{k+1} > 2(k+1)+1 = 2k+3$

$2^{k+1} > 2k+3$

$2^k \cdot 2^1 > 2k+3$

since $2^k > 2k+1$ (assumption) then $2^k \cdot 2 > (2k+1)\cdot 2$

show $2(2k+1) > 2k+3$

$2k+2k+2 > 2k+3$

for all $k \in \mathbb{Z}^+$, it's true that $2k+2 > 3$

thus, must be true that $2k+2k+2 > 2k+3$

and since $2^k \cdot 2 > 2k+2k+2 = 2(2k+1)$

then must be true that $2^{k+1} > 2k+3$

So, result is true for $n=k+1$ when $n=k$. Since result is true for $n=3$, then by principle of mathematical induction the statement is true for all $n \geq 3$, $n \in \mathbb{Z}$.

8. [Maximum mark: 7]

Solve for x in each of the following equations:

(a) $\log_2\left(5x^2 - x - 2\right) = 2 + 2\log_2 x$. [3]

(b) $3^{x+1} = 2^{2-x}$. Express the answer in the form $\dfrac{\ln a}{\ln b}$, $a, b \in \mathbb{Q}$. [4]

(a) $\log_2(5x^2 - x - 2) = \log_2 4 + \log_2 x^2$

$\log_2(5x^2 - x - 2) = \log_2 4x^2$

thus, $5x^2 - x - 2 = 4x^2 \Rightarrow x^2 - x - 2 = 0$

$(x-2)(x+1) = 0 \rightarrow \underline{\underline{x = 2}}$ or $\cancel{x = -1}$

not allowed

(b) $3^{x+1} = 2^{2-x}$

$\ln\left(3^{x+1}\right) = \ln\left(2^{2-x}\right)$

$(x+1)\ln 3 = (2-x)\ln 2$

$x\ln 3 + \ln 3 = 2\ln 2 - x\ln 2$

$x\ln 3 + x\ln 2 = \ln 2^2 - \ln 3$

$x(\ln 3 + \ln 2) = \ln 4 - \ln 3$

$x\ln 6 = \ln \frac{4}{3}$

$$\underline{\underline{x = \frac{\ln \frac{4}{3}}{\ln 6}}}$$

9. [Maximum mark: 6]

The coefficients of x^2 in the expansions $(1+x)^{2n}$ and $(1+15x^2)^n$ are equal. Given that n is a positive integer, find the value of n.

$$(a+b)^n = \sum_{r=0}^{n} \binom{n}{r} a^{n-r} b^r$$

expressions for x^2 terms in each expansion

$(1+x)^{2n}$: x^2 term $\binom{2n}{2}(1)^{2n-2}(x)^2 = \binom{2n}{2}x^2$

$(1+15x^2)^n$: x^2 term $\binom{n}{1}(1)^{n-1}(15x^2)^1 = \binom{n}{1}15x^2$

equating the coefficients

$$\binom{2n}{2} = 15\binom{n}{1}$$

$$\frac{(2n)!}{2(2n-2)!} = 15n$$

$$2n(2n-1) = 30n$$

$$4n^2 = 32n$$

$$\underline{\underline{n = 8}}$$

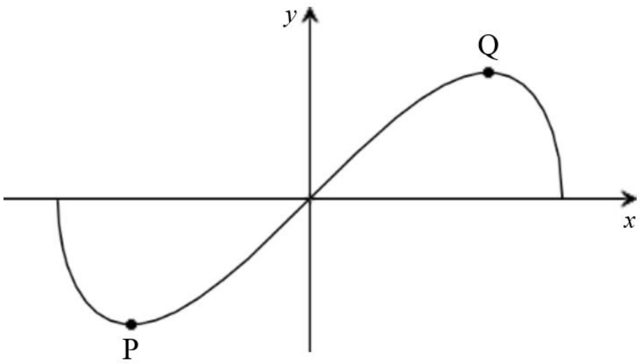
Do **not** write solutions on this page.

Section B (51 marks)

Answer **all** the questions on the answer sheets provided. Please start each question on a new page.

10. [Maximum mark: 24]

The diagram shows the graph of the function defined by $f(x) = x\sqrt{1-x^2}$, $-1 \le x \le 1$.



The function has a minimum at the point P and a maximum at point Q.

(a) Show that $f'(x) = \dfrac{1-2x^2}{\sqrt{1-x^2}}$. [4]

(b) Find the coordinates of P, and the coordinates of Q. [4]

(c) Find the total area enclosed by the graph of f and the x-axis. [5]

(d) The graph of f is rotated 2π radians about the x-axis, forming a solid.
Show that the total volume of this solid is $\dfrac{4\pi}{15}$. [5]

The function g is defined as $g(x) = 2f(x-3)$.

(e) Determine the domain and the range of g. [4]

(f) Another solid is formed when the graph of g is rotated 2π radians about the x-axis.
Write down the total volume of this solid. [2]

Do **not** write solutions on this page.

11. [Maximum mark: 17]

Consider the points $\mathrm{A}(8, -4, 5)$, $\mathrm{B}(5, -3, 4)$ and $\mathrm{C}(3, -2, 5)$.

(a) Find the vector $\overrightarrow{\mathrm{AC}} \times \overrightarrow{\mathrm{AB}}$. [4]

(b) Determine the area of triangle ABC. [3]

(c) Plane Π_1 contains triangle ABC. Show that a Cartesian equation for Π_1 is $2x+5y-z=-9$ [3]

A second plane Π_2 is defined by the Cartesian equation $\Pi_2 :\ x+by+cz=-6$, where b and c are constants. Plane Π_2 is perpendicular to plane Π_1 and the two planes intersect at a line with the Cartesian equation $\dfrac{x+1}{-16}=\dfrac{y+1}{5}=\dfrac{z-2}{-7}$.

(d) Find the value of b, and the value of c. [4]

A third plane, Π_3, is defined by the Cartesian equation $\Pi_3 :\ x+2y-2z=9$.

(e) Given that Π_1, Π_2 and Π_3 intersect at point P, find the coordinates of P. [3]

12. [Maximum mark: 13]

Consider the complex number $w=\cos\theta+\mathrm{i}\sin\theta$.

(a) Show that $w^n+\dfrac{1}{w^n}=2\cos n\theta$ where $n\in\mathbb{Z}^+$. [3]

(b) Hence, write down an expression, in terms of $\cos\theta$, for $\left(w+\dfrac{1}{w}\right)^5$. [1]

(c) Show that $\cos^5\theta=\dfrac{1}{16}\left(\cos 5\theta+5\cos 3\theta+10\cos\theta\right)$. [4]

(d) Hence, find all the solutions of $\cos 5\theta+5\cos 3\theta+12\cos\theta=0$ in the interval $0\le\theta<2\pi$. [5]

10. (a) $f(x) = x(1-x^2)^{\frac{1}{2}}$

$$f'(x) = (1-x^2)^{\frac{1}{2}} + x\left[\tfrac{1}{2}(1-x^2)^{-\frac{1}{2}}(-2x)\right]$$

$$= (1-x^2)^{\frac{1}{2}} - x^2(1-x^2)^{-\frac{1}{2}}$$

$$= (1-x^2)^{-\frac{1}{2}}\left[(1-x^2)^1 - x^2\right]$$

$$= (1-x^2)^{-\frac{1}{2}}(1-2x^2)$$

$$f'(x) = \frac{1-2x^2}{\sqrt{1-x^2}} \qquad \text{Q.E.D.}$$

(b) $f'(x) = \dfrac{1-2x^2}{\sqrt{1-x^2}} = 0 \rightarrow 1-2x^2 = 0 \rightarrow x^2 = \frac{1}{2} \rightarrow x = \pm\frac{\sqrt{2}}{2}$

$$f\left(\tfrac{\sqrt{2}}{2}\right) = \tfrac{\sqrt{2}}{2}\sqrt{1-\left(\tfrac{\sqrt{2}}{2}\right)^2} = \tfrac{\sqrt{2}}{2}\sqrt{\tfrac{1}{2}} = \tfrac{\sqrt{2}}{2}\cdot\tfrac{\sqrt{2}}{2} = \tfrac{1}{2}$$

$$f\left(-\tfrac{\sqrt{2}}{2}\right) = -\tfrac{\sqrt{2}}{2}\sqrt{1-\left(-\tfrac{\sqrt{2}}{2}\right)^2} = -\tfrac{\sqrt{2}}{2}\cdot\tfrac{\sqrt{2}}{2} = -\tfrac{1}{2}$$

thus, $P\left(\frac{\sqrt{2}}{2}, \frac{1}{2}\right)$ and $Q\left(-\frac{\sqrt{2}}{2}, -\frac{1}{2}\right)$

(c) to find total enclosed area, find area of region from $x=0$ to $x=1$ and double it – because of symmetry of graph

$$\text{area} = 2\int_0^1 x\sqrt{1-x^2}\,dx$$

$$= 2\left[-\tfrac{1}{3}\sqrt{(1-x^2)^3}\right]_0^1$$

$$= -\tfrac{2}{3}\left[0-1\right] = \frac{2}{3}$$

find $\int x\sqrt{1-x^2}\,dx$

let $u = 1-x^2$ then $du = -2x\,dx$

$-\frac{1}{2}du = x\,dx$

$$\int x\sqrt{1-x^2}\,dx = \int\left(-\tfrac{1}{2}du\right)\sqrt{u} = -\tfrac{1}{2}\int u^{\frac{1}{2}}du$$

$$= -\tfrac{1}{2}\left(\tfrac{2}{3}u^{\frac{3}{2}}\right) = -\tfrac{1}{3}u^{3/2} = -\tfrac{1}{3}\sqrt{(1-x^2)^3}$$

[Q10 worked solution continued on next page]

10. *[continued]*

(d) double the volume found from rotating the region from $x=0$ to $x=1$

$$\text{volume} = 2\pi\int_0^1 \left(x\sqrt{1-x^2}\right)^2 dx$$

$$= 2\pi\int_0^1 x^2(1-x^2)\,dx = 2\pi\int_0^1 (x^2-x^4)\,dx$$

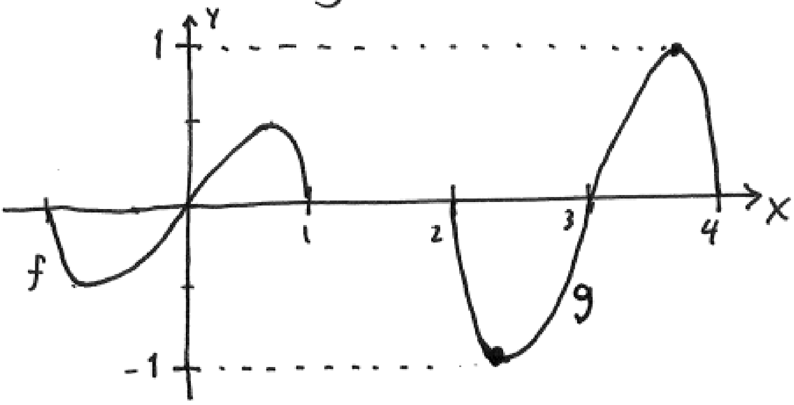
$$= 2\pi\left[\frac{1}{3}x^3 - \frac{1}{5}x^5\right]_0^1 = 2\pi\left[\left(\frac{1}{3}-\frac{1}{5}\right)-0\right]$$

$$= 2\pi\left(\frac{5}{15}-\frac{3}{15}\right) = 2\pi\left(\frac{2}{15}\right) = \frac{4\pi}{15} \quad \text{Q.E.D.}$$

(e) graph of g can be found by translating graph of f 3 units to the right and stretching it vertically by a factor of 2

domain of g: $2 \le x \le 4$

range of g: $-1 \le y \le 1$



(f) $\text{volume} = 2\pi\int_3^4 \left(2x\sqrt{1-x^2}\right)^2 dx = 2\pi\int_3^4 4\left(x\sqrt{1-x^2}\right)^2 dx$

thus, volume $= 4\left(\frac{4\pi}{15}\right) = \frac{16\pi}{15}$

result from (d)

11. (a) $\vec{AC} = \begin{pmatrix} 3-8 \\ -2+4 \\ 5-5 \end{pmatrix} = \begin{pmatrix} -5 \\ 2 \\ 0 \end{pmatrix}$ $\quad \vec{AB} = \begin{pmatrix} 5-8 \\ -3+4 \\ 4-5 \end{pmatrix} = \begin{pmatrix} -3 \\ 1 \\ -1 \end{pmatrix}$

$\vec{AC} \times \vec{AB} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -5 & 2 & 0 \\ -3 & 1 & -1 \end{vmatrix} = \vec{i}\begin{vmatrix} 2 & 0 \\ 1 & -1 \end{vmatrix} - \vec{j}\begin{vmatrix} -5 & 0 \\ -3 & -1 \end{vmatrix} + \vec{k}\begin{vmatrix} -5 & 2 \\ -3 & 1 \end{vmatrix} = \underline{\underline{-2\vec{i} - 5\vec{j} + \vec{k}}}$

(b) area $\Delta ABC = \frac{1}{2}\left|\vec{AC} \times \vec{AB}\right| = \frac{1}{2}\sqrt{(-2)^2 + (-5)^2 + 1^2} = \underline{\underline{\frac{1}{2}\sqrt{30}}}$

(c) $\vec{AC} \times \vec{AB}$ is a normal vector for plane Π_1 containing ΔABC

thus, for plane Π_1 $\vec{n} = \begin{pmatrix} -2 \\ -5 \\ 1 \end{pmatrix}$ or $\vec{n} = \begin{pmatrix} 2 \\ 5 \\ -1 \end{pmatrix}$

equation for plane Π_1 is $2x + 5y - z = d$

substitute in a point; such as $C(3,-2,5)$

$2(3) + 5(-2) - 5 = 6 - 10 - 5 = -9$

therefore, Cartesian equation for Π_1 is $2x + 5y - z = -9$

Q.E.D.

(d) since planes Π_1 and Π_2 are perpendicular, then their normal vectors will be perpendicular; also both normal vectors will be perpendicular to the direction vector of the line of intersection of Π_1 and Π_2

$\vec{n}_1 = \begin{pmatrix} 2 \\ 5 \\ -1 \end{pmatrix}$ and $\vec{n}_2 = \begin{pmatrix} 1 \\ b \\ c \end{pmatrix}$; $\vec{n}_1 \cdot \vec{n}_2 = \begin{pmatrix} 2 \\ 5 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ b \\ c \end{pmatrix} = 0$

$2 + 5b - c = 0$

direction vector for line $\frac{x+1}{-16} = \frac{y+1}{5} = \frac{z-2}{-7}$ is $\begin{pmatrix} -16 \\ 5 \\ -7 \end{pmatrix}$

$\begin{pmatrix} 1 \\ b \\ c \end{pmatrix} \cdot \begin{pmatrix} -16 \\ 5 \\ -7 \end{pmatrix} = 0 \rightarrow -16 + 5b - 7c = 0$

$\begin{cases} (1)\ 5b - c = -2 \\ (2)\ 5b - 7c = 16 \end{cases}$ $\quad (2)-(1):\ -6c = 18 \rightarrow \underline{\underline{c = -3}}$

$5b - (-3) = -2 \rightarrow 5b = -5 \rightarrow \underline{\underline{b = -1}}$

[Q11 worked solution continued on next page]

11. *[continued]*

(e) planes Π_1 and Π_2 intersect at line $\dfrac{x+1}{-16}=\dfrac{y+1}{5}=\dfrac{z-2}{-7}$, so find the point of intersection between this line and the plane Π_3

$$\frac{x+1}{-16}=\frac{y+1}{5}=\frac{z-2}{-7}=\lambda \Rightarrow \begin{array}{c} x=-1-16\lambda \\ y=-1+5\lambda \\ z=2-7\lambda \end{array}$$

Substituting into equation for Π_3:

$$(-1-16\lambda)+2(-1+5\lambda)-2(2-7\lambda)=9 \Rightarrow -7+8\lambda=9 \Rightarrow 8\lambda=16 \Rightarrow \lambda=2$$

Substituting $\lambda=2$ into parametric equations of the line:

$$\begin{array}{c} x=-1-16(2) \\ y=-1+5(2) \\ z=2-7(2) \end{array} \Rightarrow \begin{array}{c} x=-1-32 \\ y=-1+10 \\ z=2-14 \end{array} \Rightarrow \begin{array}{c} x=-33 \\ y=9 \\ z=-12 \end{array}$$

Hence, coordinates of P are $(-33,\ 9,\ -12)$

[Q12 worked solution on next page]

12. (a) $w^n + \frac{1}{w^n} = w^n + w^{-n}$

de Moivre's theorem:

$w^n = \cos n\theta + i\sin n\theta$

$w^{-n} = \cos(-n\theta) + i\sin(-n\theta) = \cos n\theta - i\sin n\theta$

$[\cos(-\theta) = \cos\theta$ and $\sin(-\theta) = -\sin\theta]$

thus, $w^n + \frac{1}{w^n} = (\cos n\theta + i\sin n\theta) + (\cos n\theta - i\sin n\theta)$

$= 2\cos n\theta$ Q.E.D.

(b) $w + \frac{1}{w} = w^1 + \frac{1}{w^1} = 2\cos(1\cdot\theta) = 2\cos\theta$

thus, $\left(w + \frac{1}{w}\right)^5 = (2\cos\theta)^5 = \underline{\underline{32\cos^5\theta}}$

(c) from (b): $(w + w^{-1})^5 = 32\cos^5\theta$

expand binomial: $(w + w^{-1})^5 = w^5 + 5w^3 + 10w + 10w^{-1} + 5w^{-3} + w^{-5}$

$32\cos^5\theta = (w^5 + w^{-5}) + 5(w^3 + w^{-3}) + 10(w + w^{-1})$

$= 2\cos 5\theta + 10\cos 3\theta + 20\cos\theta$

$\cos^5\theta = \frac{1}{16}(\cos 5\theta + 5\cos 3\theta + 10\cos\theta)$ Q.E.D.

(d) $\cos 5\theta + 5\cos 3\theta + 12\cos\theta = 0$ [re-arrange to match this]

$\cos 5\theta + 5\cos 3\theta + 10\cos\theta = -2\cos\theta$

from result in (b): $16\cos^5\theta = -2\cos\theta$

$2\cos\theta(8\cos^4\theta + 1) = 0$

$8\cos^4\theta + 1 = 0$ has no solution

$\cos\theta = 0$

$\underline{\underline{\theta = \frac{\pi}{2}, \frac{3\pi}{2}}}$